Thesis Topics

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Pick your problem!

You are welcome to chose your favourite problem – probably this is the best option for you as you should really work on something you're enthusiastic about. As long as it is from one of classical areas of Graph Theory, I will most likely be happy to work with you. A nice list of interesting open problems can be found here.

`http://www.openproblemgarden.org/category/graph_theory`

Take your time, pick your problem (or a couple), send me corresponding links, and allow a few days for me to sketch a plan for your thesis. In the best case we both learn something exciting.

Below are two very different topics which are also interesting to me.

Cycle Regularity and Petersen-related Families

The *generalized Petersen graphs*, introduced by Coxeter et al. [1950] and named by Watkins [1969], form a very interesting family of trivalent graphs that can be described by only two integer parameters. They include Hamiltonian and non-Hamiltonian graphs, bipartite and non-bipartite graphs, vertex-transitive and non-vertex-transitive graphs, Cayley and non-Cayley graphs, arc-transitive graphs and non-arc transitive graphs, graphs of girth $3, 4, 5, 6, 7$ or 8. Their generalization to I-graphs does not introduce any new vertex-transitive graphs but it contains also non-connected graphs and has in special cases unexpected symmetries.

Various aspects of the structure of the mentioned family has been observed. Examples include their linear recognition algorithm for generalized Petersen graphs [Krnc and Wilson, 2020], identifying those that are Hamiltonian [Alspach, 1983] or Cayley [Saražin, 1997, Nedela and Škoviera, 1995], or finding their automorphism group [Steimle and Staton, 2009, Petkovšek and Zakrajšek, 2009, Horvat et al., 2012]. Also, a related generalization to I-graphs has been introduced in the Foster census [Bouwer et al., 1988], and further studied by Boben et al. [2005], Klobas and Krnc [2020].

Together with some of my collegues (N. Klobas, T. Pisanski, R. J Wilson) we already established some structural results from the area, and identified interesting further research directions one can take; those would the subject of the candidate's thesis.

For this project, knowledge in graph theory, as well as topological and algebraic aspects of graph theory is appreciated.

Distributed Consensus on Large Randomised Networks

and it's role in cryptocurrencies...

The consensus problem may be defined as the following simple process: In the initial graph, every vertex is assigned one of k opinions. The goal is for all nodes to reach the consensus as efficiently (number of time-steps needed and number of overall message transmissions) as possible. Here, one should consider several models depending on the communication restriction as well as network topology.

Throughout the history of consensus protocols, several basic ideas have been proposed. Among these, the classic pull and push algorithms permit a particularly nice behavior, which may be modeled as Markov process. Other approaches require probabilistic tools such as martingales and corresponding concentration inequalities.

The mentioned problems play a super-important role in synchronization protocols and represent a cornerstone in an emerging technologies such as blockchain. Indeed, many crucial advantages or disadvantages of the cryptocurrencies lie in understanding the underlying consensus problem. Among possible directions are:

- The beauty of Consensus - getting in touch with core ideas from the field. Proposing new algorithms.
- Blockchain topics - its theoretical aspects including fundamental role of consensus and distributed algorithms. A comparison of different consensus solutions in the area of cryptocurrencies.
- Simulation topics - implementing an environment where one can analyse various protocols for distributed computing.

This topic requires knowledge on the probability and random graph theory (suggested books are [Mitzenmacher and Upfal, 2005] and [Bollobás, 1998]).

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